The newsvendor problem with GHG emissions from waste disposal under cap-and-trade regulation

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Abstract

We study a newsvendor problem with greenhouse gas (GHG) emissions at the disposal stage regulated by a cap-and-trade policy. We first consider a synchronous version of the problem where the replenishment period and the emission assessment period are the same. For this problem, we propose simple algorithms to derive the optimal quantity and cost for both continuous and discrete demand distributions. We show that ignoring the carbon tax may lead to large overcost, especially when the GHG quota is small. For a normally distributed demand, we also show that the demand standard deviation has a non-linear and non-monotonic effect on the order quantity. We then investigate an asynchronous version of the problem where replenishment decisions occur at a higher frequency than the quota allocation. We formulate this lot sizing problem as a Markov decision process and solve it with a dynamic programming algorithm whose complexity is assessed. Through a numerical experimentation and theoretical results, we demonstrate how and when the flexibility of the optimal dynamic replenishment decisions allows to reduce costs. When approaching the end of the time horizon, the uncertainty reduction allows the decision maker to adopt a riskier policy and order larger quantities.

Keywords : Newsvendor, GHG emissions, Cap-and-trade.

1 Introduction

1.1 Background and motivation

With growing concerns about the burden of human activity on local and global ecosystems, industry leaders and researchers are increasingly looking for ways to reduce said ecological footprint [GSIA, 2017, Chen et al., 2022]. Almost all fields of human activity are concerned,

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from agriculture to manufacturing, from tourism to transportation. Numerous new operation research models are developed to meet this challenge.

In an effort to face what appears to be the most important challenge currently facing humanity, regulations have been put in place to reduce emissions of GHG in general, and of carbon dioxide (CO_2) in particular. The so called cap-and-trade mechanism allocates large organizations, either in the private (e.g. industrial) or public (e.g. countries or municipalities) sectors, a certain allowance (cap, or quota) to emit GHG [Schmalensee and Stavins, 2017]. Above this limit, a penalty is enforced. A player that successfully reduces its emissions and doesn't meet its quota can sell the surplus allowance to another one that wishes in doing so, not to pay said penalty. GHG emissions are measured usually on a yearly basis in units of equivalent tons of emitted CO_2 following a predefined universal protocol depending on the sector at hand.

In 2016, it was calculated that the management of solid waste produced approximately 1.6 billion tonnes of carbon dioxide–equivalent greenhouse gas emissions, which accounts for roughly 5% of the world's total emissions. Without improvement in this area, it is predicted that solid waste-related emissions will surge to 2.6 billion tonnes of $\mathrm{CO_2}$-equivalent by the year 2050 [Kaza et al., 2018]. For some sectors of activity, GHG emissions are mostly emitted during the disposal stage. This is consistent, for example, with industries that mostly acquire and sell perishable agricultural products or food products. Landfilling organic or mixed waste tend to emit considerable amounts of $\mathrm{CO_2}$, but also of methane, which is 25 times more potent in terms of greenhouse effect [Jha et al., 2008, Nevrlỳ et al., 2019, Nordahl et al., 2020, Sánchez et al., 2015]). The disposal stage can also represent the main vector for GHG emissions when industries make extensive use of renewable energy at the production stage [Jin et al., 2018].

1.2 Research questions

To include the GHG cap-and-trade constraint in the logistics of enterprises, various well known operational models have been updated, among which we find the newsvendor problem (NVP). The present work considers a NVP under a cap-and-trade regulation on GHG emissions, for the case in which emissions occur at the disposal stage. We first study a problem, denoted synchronous D-NVP (D for disposal), which describes the case in which the demand realization period and the GHG emission assessment period are the same. In practice, the disposal process and corresponding quota, dictated either by a regulating authority (e.g. GHG emmissions) or by a disposal contractor (e.g. landfill) is typically conducted on a different time basis than the replenishment decisions. This is specially true for perishable goods, which are purchased on a much shorter periodical basis (e.g. daily or weekly) than the typical GHG emissions accounting (yearly). We model this reality by introducing a second problem, denoted asynchronous D-NVP, in which replenishment decisions are made more frequently than the emission assessment. At each period, replenishment takes place following a regular NVP scheme. Products are perishable and thus unsold items are disposed of. But it is only at the end of the multi-period horizon that the GHG emissions are accounted for. Only if this quantity exceeds the quota does the company pay a tax per item disposed of in excess of said quota, or receives a reward per unit of unused quota (cap-and-trade).

The research questions we address are as follows. We explore the structure of the optimal policies for both synchronous D-NVP and asynchronous D-NVP. We also study the sensitivity of the optimal policy and cost to key parameters of the problem. Finally, we study the effect of the replenishment frequency (versus emission assessment frequency) on the efficiency of the supply chain.

1.3 Main contributions

Disposal in general and landfilling in particular are major contributors to GHG emissions [Lou and Nair, 2009, Kumar and Sharma, 2014]. However, to the best of our knowledge, this has not yet been considered in the inventory management literature in the context of a cap-and-trade emission regulation. The main contribution of the present work is to consider a NVP with GHG emissions at the disposal stage under a cap-and-trade emission regulation. The other emission regulation mechanisms (strict cap, linear carbon tax and cap-and-offset) can be seen as special cases of the cap-and-trade scheme, as pointed out by Chen et al. [2013].

We first consider a synchronous problem in which the demand realization period and the GHG emission assessment period are the same. This problem is closely related to Song and Leng [2012], but while they consider emissions associated with ordering, our model focuses on emission generated at the disposal stage. It is also related to Qu et al. [2021] who consider emissions at the disposal stage with a linear carbon tax, (see Section 2.3). We consider a more general cap-and-trade mechanism which is far more complex to analyze and can not be reduced to a standard NVP. For the synchronous problem, we present a simple algorithm to compute the optimal order quantity for continuous and discrete demand distributions. We obtain additional results for specific demand distributions like the normal distribution. We also derive managerial insights, including the cost of ignoring the carbon regulation or the effect of demand variability on operational costs. We show that ignoring the carbon tax may lead to large overcost, especially when the GHG quota is small. We also demonstrate that the demand variability has a more complex effect in the D-NVP than in the standard NVP.

Secondly, we address the more general asynchronous problem in which replenishment decisions are made more frequently than the emission assessment. While closely related to the problems presented in Gong and Zhou [2013] and Zhang et al. [2016], the asynchronous problem presented in this work differs from theirs in two aspects, namely, that we assume that GHG emissions are generated by disposal (versus production) and that inventory is not transferable from one period to another. We formulate the asynchronous problem as a Markov decision process where the system state is, at each period, the unused emission quota. We use a dynamic programming algorithm (whose complexity is assessed) to compute the optimal policy and the optimal cost. We establish that the optimal cost is decreasing with unused quota and time. Finally, we demonstrate the superiority of a policy that takes advantage of the flexibility offered by a multi-period dynamic policy over simpler rules. Incidentally, allowing the company to adjust more closely its activity to the planned cap also improves the predictability of overall emissions to the regulator.

In the next section we provide an overview of the literature in the relevant fields. In Section 3, we study the synchronous D-NVP. Then we consider the asynchronous D-NVP in Section 4.

2 Literature review

In the present work, the gap we intend to address lies in the very special structure found in cap-and-trade regulation for GHG emissions, when applied to the disposal stage of NVPs. Following this, we review the literature related to inventory models in which a carbon tax is levied and/or a limit on emissions is imposed.

For a review on the broader field of sustainable operations management, the reader is addressed to Hong et al. [2022]. Their review is comprehensive on a wide range of topics including supplier selection, green design, production and logistics including among other disposal and emissions management. For more specific overviews of the literature on low-carbon supply chain management, including on the carbon legislation, social aspects of the problem, and modeling methodologies, the reader is addressed to Chelly et al. [2019] and Zhou et al. [2021].

For an overview of how the cap-and-trade mechanism is integrated into various operational, financial or regulatory considerations in recent years, the reader is addressed among others to Chen et al. [2021], Li et al. [2021], Liu et al. [2021], Chen et al. [2020], and Schmalensee and Stavins [2017]

2.1 EOQ models

We first review the literature dealing with an EOQ setting where a firm faces a continuous and constant demand over time, as in the seminal work of Harris [1913]. Hua et al. [2011] consider an EOQ model subject to cap-and-trade regulation where emissions are generated by ordering and inventory holding. The penalty incurred per unit emitted above the cap is equal to the reward on each unit saved from the cap. This assumption greatly simplifies the analysis. They show that the optimal order quantity is between the standard optimal order quantity (without accounting for emissions) and the order size that minimizes carbon emissions. They also show that the cap-and-trade mechanism reduces carbon emissions while increasing the total cost.

Chen et al. [2013] study several variants of Hua et al. [2011]. The carbon emissions are associated with ordering, inventory holding but also with production/purchasing. They consider four mechanisms : strict cap (emissions must no exceed a cap), carbon tax (penalty per unit of carbon emitted), cap-and-offset (tax only emissions that exceed a certain threshold) and cap-and-trade (penalty/reward per unit of emission above/below the cap). For the first three mechanisms (strict cap, carbon tax and cap-and-offset), they derive the optimal quantity and cost. They also remark that the first three schemes can be viewed as special cases of the cap-and-trade mechanism. For instance, when the reward equals the penalty, the cap-and-trade problem is similar to the one with a carbon tax. They also show that "the opportunity for reducing carbon emissions via operational adjustments exists whenever the operational drivers of emissions are different from the operational drivers of costs."

He et al. [2015] study more in detail the cap-and-trade mechanism discussed in Chen et al. [2013]. They derive the optimal quantity when the reward is different from the penalty. They also study the carbon tax scheme and reach results similar to Chen et al. [2013]. In addition to considering the effects of regulation on the lot size, decisions also include the level of permit trading (buying or selling) for the cap-and-trade case. We can also mention

Bouchery et al. [2012], Arslan and Turkay [2013] that have investigated similar problems or Toptal et al. [2014] that consider the joint decisions on inventory replenishment and emission reduction investments.

2.2 Deterministic lot-sizing models

Another stream of literature considers a lot-sizing setting with a deterministic demand per period that varies over time. Benjaafar et al. [2012] investigate several uncapacitated lot-sizing problems that incorporate carbon cap/tax. They first consider a single firm that must determine when and how much to order over a planning horizon with multiple periods and known demand. The cost structure includes ordering, holding and backorder costs. The carbon emissions are associated with ordering, inventory holding and production/purchasing. They formulate the decision problem with a MILP (mixed integer linear program) for four mechanisms : strict cap, carbon tax, cap-and-offset and cap-and-trade. They also extend the approach to a serial supply chain consisting of N different firms that either collaborate or not. Among the presented observations, we cite here two important ones: "Tighter caps on emissions can paradoxically lead to higher total emissions" and "Imposing supply chain-wide emission caps leads to lower emissions at lower costs; it also increases the value of collaboration".

Some research papers study the complexity of the economic lot-sizing problem with carbon cap/tax (see e.g. Retel Helmrich et al. [2012], Akbalik and Rapine [2014], Helmrich et al. [2015]). In particular, it is shown that the single item uncapacitated lot-sizing problem with an emission capacity constraint is NP-hard.

Other research papers consider multiple modes of production or transport. Absi et al. [2013] consider a multi-sourcing lot-sizing problem with an emission constraint that can be defined per period, on the whole horizon or over a rolling horizon. They show that the problem with a constraint per period can be polynomially solved while the others are NP-hard. Absi et al. [2016] consider the more general setting of a fixed carbon emission associated with each mode, in addition to its unit carbon emission. They show that the problem with a constraint per period is then NP-hard. Hong et al. [2016] obtain additional results in the special case of dual sourcing (green or regular). Phouratsamay and Cheng [2019] add inventory bounds to the problem. Palak et al. [2014] consider an application to the biofuel supply chain.

Finally, we mention some variants that also consider carbon emission constraints. Wu et al. [2018] consider a multi-item lot sizing problem with parallel machines. Azadnia et al. [2015] and Bodaghi et al. [2018] study a lot sizing problem with sustainable supplier selection. Vaez et al. [2019] present a bi-objective model to solve a lot-sizing and scheduling problem simultaneously that would maximize the total profit and minimize CO_2 emissions. Shaw et al. [2023] include, in addition to carbon footprint, other dimensions such as water footprint, solid and liquid waste, etc. Zouadi et al. [2018] study a system with both regular manufacturing and remanufacturing processes.

2.3 Newsvendor models

Another class of canonical problems that has been reformulated to incorporate carbon tax/cap is the NVP. The NVP strives to determine the quantity to order for a single acquisition and selling period, under a random demand with known distribution (see e.g. Qin et al. [2011] for a literature review on NVP). Song and Leng [2012] study a NVP where carbon emissions occur at the production stage. They consider several emission regulations (mandatory capacity, taxation and cap-and-trade) and derive the optimal policy for each of them. Under the cap-and-trade regulation, the penalty paid for each unit exceeding the production quota and the reward for each unused unit of the quota are different, which adds complexity over the standard NVP. Qu et al. [2021] propose a more general emission model where GHG emissions are accounted at different stages (production, sales, disposal, and returns). They consider a cap-and-trade mechanism in which the per unit reward for not reaching the emission quota is equal to the penalty in case of an excursion from said quota. With this last assumption, the cap-and-trade mechanism turns into a simple linear carbon tax mechanism and therefore the optimal order quantity is independent of the allocated quota.

Zhang and Xu [2013] combine a carbon cap-and-trade mechanism into a single-period capacitated multi-item production planning problem. They analyze the sensitivity of their solution to carbon price and carbon cap. In Arıkan and Jammernegg [2014] as well as in Choi [2013], a carbon tax is considered in a dual ordering single period system typical of the fashion industry. In both cases the carbon emissions are simply added to the cost of production, and the additional emissions due to disposal are blended in by the use of the estimated proportion of leftovers. Rosič and Jammernegg [2013] consider an NVP with dual sources, a primary one with low cost and long lead time, and a secondary with short lead time and a higher purchasing price subject to taxing and trading on emissions from transport.

Heydari and Mirzajani [2021] show how the coordination of a manufacturing supply chain under a revenue sharing agreement allows to optimize both profitability and sustainability. Li et al. [2019] explore the impact of two different carbon permit allocations mechanisms on the emissions of a two-echelon supply chain for different types of emission reduction incentive between the retailer and the supplier.

Bai and Chen [2016] formalize a robust dual source NVP with cap-limited carbon emissions generated by each of the two order types. Qi et al. [2021] propose a risk management approach to reach a dual decision on order quantity and emission levels under cap-and-trade regulation with emission sensitive demand driven by environmentally aware customers.

2.4 Stochastic lot-sizing

Finally, we review the literature dealing with a lot-sizing setting where a firm faces a stochastic demand in each time period. Gong and Zhou [2013] consider a manufacturer that has access to both green and regular production technologies with GHG emissions linear with produced quantities. At each period a series of decisions are made on the technology to be used (characterized by its cost and its level of emissions), the quantities to be produced, and the emission allowances to be traded (bought or sold). At the end of the horizon, the total cost is assessed as the sum of all periodic costs pertaining to production, emission trading, holding and backlogging, with the addition, when applicable, of a one time cost induced by products overage and a heavy penalty for emission excursions. The authors provide structural properties of the optimal policy, which includes trading and production decisions. While Gong and Zhou [2013] address the dual-technology problem, Zhang et al. [2016] consider a more general problem with any number of technologies. Ma et al. [2018] study a problem combining producer choice and lot sizing. In Shi et al. [2023], a procurement commitment contract between a retail giant and a capital constrained supplier is considered in the context of a low carbon supply chain where the supplier can invest in carbon emissions reduction. They present a Stackelberg game in which the retailer is the leader and the supplier the follower.

We can also mention a related stream of literature addressing the hydro-thermal scheduling problem under GHG emissions constraints (see e.g. Rebennack et al. [2011] and Rebennack [2014]).

Table 1 gives an overview of the publications most related to our work.

Article	Model type	Demand	Source of emissions	Emission regulation
Hua et al. [2011]	EOQ	Deterministic	Ordering, production, inventory	Cap-and-trade
Chen et al. [2013]	EOQ	Deterministic	Ordering, production, Inventory	Tax, cap, cap-and-offset, cap-and-trade
He et al. [2015]	EOQ	Deterministic	Ordering, production, inventory	Tax, cap-and-offset, cap-and-trade
Benjaafar et al. [2012]	Lot-sizing	Deterministic	Ordering, production, inventory	Tax, cap, cap-and-offset, cap-and-trade
Akbalik and Rapine [2014]	Lot-sizing	Deterministic	Ordering, production, inventory	Cap-and-trade
Song and Leng [2012]	NVP	Stochastic	Production	Cap, tax, cap-and-trade
Qu et al. [2021]	NVP	Stochastic	Production, sales, disposal, returns	Tax
The present paper	NVP, lot-sizing	Stochastic	Disposal	Cap-and-trade

Table 1: Overview of most relevant publications

3 The synchronous D-NVP

In this section, we first formalize the synchronous problem. Then we present some preliminary results before deriving the optimal quantity. Finally, we compare the optimal solution with simple heuristics.

3.1 Assumptions and notations

Let D be the random demand with cumulative distribution (c.d.f.) function $F(\cdot)$ and expected value $E(D) = \mu$. If F is strictly increasing, we denote by $F^{-1}(\cdot)$ its inverse function. When the demand is continuous, we denote by $f(\cdot)$ its probability density function (p.d.f.) of D. When the demand is discrete, we denote by $p(\cdot)$ its probability mass function (p.m.f.).

Let $h > 0$ be the per-unit overage cost and $b > 0$ the per-unit underage cost. The company is allocated a GHG quota of $x \geq 0$. It pays a GHG tax $c_d > 0$ for each disposed unit exceeding the quota and is rewarded $r_d \geq 0$ for each unused unit of the quota (e.g. by trading emission allowance). To avoid opportunistic behavior, we assume that $c_d > r_d$ which is a standard assumption in the literature (see e.g. Gong and Zhou [2013]).

Let $v(x, q)$ be the expected cost when we order a quantity q and the allocated quota is x. We have

$$v(x, q) = G(q) + Z(x, q). \tag{1}$$

where $G(q)$ denotes the expected cost of a standard NVP and $Z(x, q)$ denotes the expected cost related to GHG emissions. The expected cost of a standard NVP is given by

$$G(q) = hE(q - D)^+ + bE(D - q)^+ \tag{2}$$

where $y^+ = \max(0, y)$ is the positive part of y. By using the property that $(q - D)^+ = (q - D) + (D - q)^+$, we can rewrite $G(q)$ as

$$G(q) = h(q - E(D)) + (h + b)E(D - q)^+ \tag{3}$$

where we only have to compute $E(D - q)^+$. The expected cost related to GHG emissions is given by

$$Z(x, q) = c_d E\left[(q - D)^+ - x\right]^+ - r_d E\left[x - (q - D)^+\right]^+. \tag{4}$$

The objective is to determine the order quantity that minimizes the expected cost $v(x, q)$. We will refer to this problem as D-NVP. The problem of minimizing $G(q)$ will be refered to as standard NVP. Let $v^*(x)$ be the optimal expected cost for quota x. We have

$$v^*(x) = \min_{q \geq 0} \quad v(x, q).$$

Since there may exist several optimal quantities when $v(x, q)$ is not strictly convex, we denote by $q^*(x)$ the smallest optimal order quantity for quota x. When x goes to infinity, the problem reduces to a standard NVP. Therefore we denote by $q^*(\infty)$ and $v^*(\infty)$ the optimal quantity and cost of a standard NVP.

We show in Appendix A that we can set the GHG reward to zero ($r_d = 0$) without loss of generality. Therefore, we make this assumption in the remainder of this article. Moreover, since $[(q - D)^+ - x]^+ = [q - D - x]^+$ when $x \geq 0$, the emission cost can be expressed in the following simpler way:

$$Z(x, q) = c_d E\left[q - D - x\right]^+. \tag{5}$$

For the reader's convenience, we summarize below the main notations that will be used in subsequent sections.

Parameters	
D	Random demand
$F(\cdot)$	Cumulative distribution function of D
$f(\cdot)$	Probability density function (for continuous random variable D)
$p(\cdot)$	Probability mass function (for discrete random variable D)
h	Overage cost
b	Underage cost
x	GHG quota
c_d	GHG tax for each unit exceeding the quota
r_d	GHG reward or each unused unit of quota

Decision variables	
q	Order quantity
$q^*(x)$	Optimal quantity

Cost functions	
$G(q)$	Expected cost of a standard NVP
$Z(x,q)$	Expected cost related to GHG emissions with order quantity q
$v(x,q)$	Total expected cost when order quantity is q
$v^*(x)$	Optimal expected cost

3.2 Preliminary results

We begin by presenting some preliminary results that will be useful to derive the optimal policy. In what follows, we study the convexity and monotonicity of the expected cost $v(x,q)$ with respect to quota x and order quantity q. Note that the following results hold for any demand distribution (either discrete or continuous).

Since $(q-D)^+$ and $(D-q)^+$ are convex in q, so are their expectations $E(q-D)^+$ and $E(D-q)^+$. It follows that $G(q)$ is also convex in q since it is the sum of two convex functions (see e.g. Porteus [2002]). Function $\gamma(x,q) = (q-D-x)^+$ is a 2-dimensional convex function. It is also non-decreasing in q and non-increasing in x. It follows that $E[q-D-x]^+$ has the same properties and so does $Z(x,q)$, given in Equation (5).

Lemma 1 (GHG emission cost) *The expected cost related to GHG emissions, $Z(x,q)$ is a convex function. It is also non-decreasing in order quantity q and non-increasing in GHG quota x.*

On one hand, $G(q)$ is convex in q and independent of x. On the other hand, $Z(x,q)$ is a convex function by Lemma 1. It follows that the total expected cost $v(x,q)$ is a convex function. By Theorem A.4 in Porteus [2002], we also have that $v^*(x) = \min_q v(x,q)$ is convex. It is also non-increasing in x

Theorem 1 (Total cost) *Cost functions $v(x,q)$ and $v^*(x)$ are convex. They are also non-increasing in GHG quota x.*

Finally, we establish two results that will be used later. The following lemma establishes that xc_d represents the maximum benefit that can be expected with a GHG quota x (versus a null quota).

Lemma 2

$$v^*(0) - xc_d \leq v^*(x) \leq v^*(0)$$

This lemma is proven in Appendix B.

We have already noticed that the problem reduces to a standard NVP when the quota x goes to infinity. The following lemma presents a stronger result. When the quota x is larger or equal than the optimal quantity of the standard NVP, the optimal quantity and cost are the same as those of a standard NVP.

Lemma 3 *If $x \geq q^*(\infty)$, then $q^*(x) = q^*(\infty)$ and $v^*(x) = v^*(\infty)$.*

The proof of this lemma is given in Appendix C.

Based on the above general results that hold for any distribution, specific properties of the problem and its solution are presented in the next section.

3.3 Optimal quantity and cost

In this section, we derive the optimal quantity and cost first for a continuous demand and then for a discrete demand. We also obtain additional results for specific distributions, including the normal distribution which is most commonly used in inventory management.

3.3.1 Continuous demand

We will now provide additional results for a continuous demand with p.d.f. f. From Equation (5), we have

$$Z(x, q) = c_d \int_{-\infty}^{q-x} (q - x - s) f(s) ds. \tag{6}$$

We can differentiate this function by using Leibniz rule :

$$\begin{aligned} \frac{\partial Z}{\partial q}(x, q) &= c_d \int_{-\infty}^{q-x} f(s) ds \\ &= c_d F(q - x). \end{aligned}$$

It is also well known for the standard NVP that $G'(q) = (h + b)F(q) - b$ (see e.g. Porteus [2002]). It follows that

$$\begin{aligned} \frac{\partial v}{\partial q}(x, q) &= G'(q) + \frac{\partial Z}{\partial q}(x, q) \\ &= (h + b)F(q) + c_d F(q - x) - b. \end{aligned}$$

On one hand, the function $v(x,q)$ is convex in q (see Theorem 1). On the other hand, we have

$$\lim_{q\to-\infty}\frac{\partial v}{\partial q}(x,q) = -b < 0,$$
$$\lim_{q\to+\infty}\frac{\partial v}{\partial q}(x,q) = h + c_d > 0.$$

Hence the minimum with respect to q is reached when the partial derivative equals zero, i.e. when $(h+b)F(q)+c_dF(q-x)=b$. We also show in Appendix D that one more unit of quota increases the optimal quantity by (at most) one more unit.

Theorem 2 (Continuous demand) *For the synchronous D-NVP with continuous demand and quota x, the smallest optimal order quantity is*

$$q^*(x) = \min\left\{q:\ (h+b)F(q)+c_dF(q-x)=b\right\}. \tag{7}$$

Moreover, we have

$$q^*(x) \le q^*(x+1) \le q^*(x)+1. \tag{8}$$

Note that the optimal quantity can be computed efficiently by a numerical method, e.g. by a bisection method, since F is a non-decreasing function.

Consider now two limit cases. When the quota is null ($x=0$), we pay the GHG tax c_d for all unsold units and the problem is equivalent to a standard NVP with overage cost $h+c_d$ and underage cost b. When the quota is infinite, no GHG tax is incurred and the problem is equivalent to a standard NVP with overage cost h and underage cost b. Let $q^*(\infty)=\lim_{x\to\infty}q^*(x)$ be the optimal quantity when the quota goes to infinity. If we assume, additionally, that F is strictly increasing, then the optimal solution is unique and we have, in the limit cases

$$q^*(0) = F^{-1}\left(\frac{b}{h+c_d+b}\right),$$
$$q^*(\infty) = F^{-1}\left(\frac{b}{h+b}\right). \tag{9}$$

As an application of Theorem 2, we derive closed-form expressions for the optimal quantity and cost for three continuous demand distributions (exponential, uniform and normal). For the exponential and uniform distributions, the results are given in appendices E.1 and E.2 respectively. The next section provides complementary results for the normal distribution and numerical examples.

3.3.2 The special case of normal distribution

In this section, we assume that the demand D follows a normal distribution $\mathcal{N}(\mu,\sigma^2)$. Let also $Z=\frac{D-\mu}{\sigma}$ be the standard normal random variable with p.d.f. $\phi(t)=\frac{1}{\sqrt{2\pi}}e^{-t^2/2}$ and c.d.f. $\Phi(t)=P(Z\le t)$. The following theorem, established in Appendix E.3, provides additional results to compute the optimal quantity and cost.

Theorem 3 (Normal distribution) *The optimal quantity and cost are given by*

$$q^*(x) = \mu + \sigma z^*(x),$$
$$v^*(x) = \sigma(h+b)\phi(z^*(x)) + c_d[\sigma\phi(w^*(x)) - x\Phi(w^*(x))],$$

where $w^*(x) = z^*(x) - \frac{x}{\sigma}$ *and* $z^*(x)$ *is the unique solution of*

$$(h+b)\Phi(z^*(x)) + c_d\Phi\left(z^*(x) - \frac{x}{\sigma}\right) = b.$$

When the quota x goes to infinity, we re-obtain the results for a standard NVP with normal demand :

$$z^*(x) \underset{x\to+\infty}{\longrightarrow} z^*(\infty) = \Phi^{-1}\left(\frac{b}{h+b}\right),$$
$$w^*(x) \underset{x\to+\infty}{\longrightarrow} -\infty,$$
$$q^*(x) \underset{x\to+\infty}{\longrightarrow} \mu + \sigma\, z^*(\infty), \tag{10}$$
$$v^*(x) \underset{x\to+\infty}{\longrightarrow} \sigma(h+b)\phi\left[z^*(\infty)\right]. \tag{11}$$

Figure 1 illustrates the effect of the quota x on the optimal quantity and cost, for several values of the GHG tax c_d. The curve with $c_d = 0$ corresponds to a standard NVP. When quota x goes to infinity, all curves converge to the standard NVP curve. We also see that the optimal order quantity $q^*(x)$ (resp. optimal cost $v^*(x)$) is non-decreasing (resp. non-increasing) with quota x. Finally, we observe that $q^*(x)$ is concave in x and that $v^*(x)$ is convex in x.

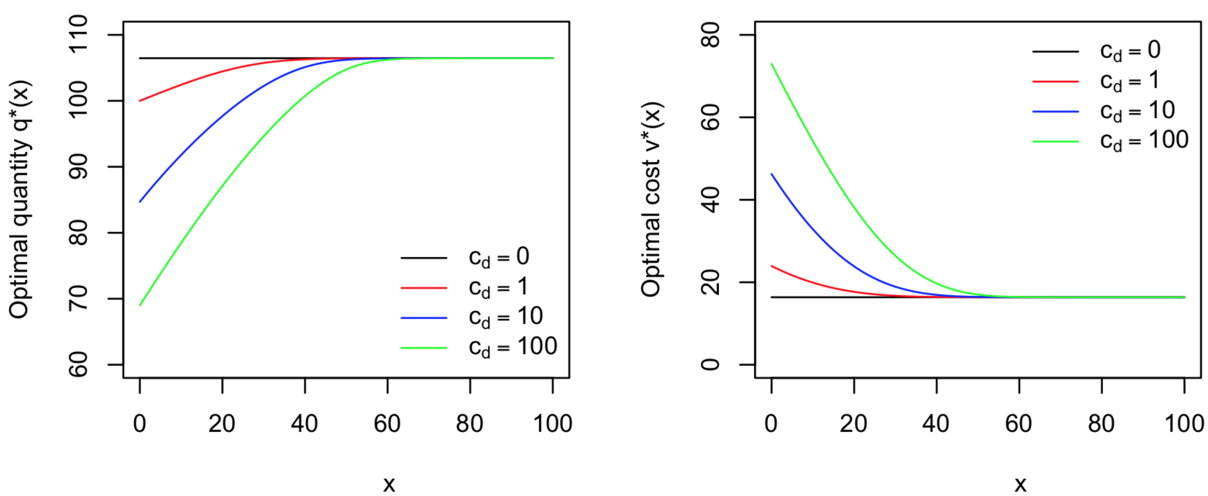

Figure 1: Effect of quota x on the optimal quantity and cost ($h = 1, b = 2, r_d = 0$, normally distributed demand with parameters $\mu = 100, \sigma = 15$)

Figure 1 Alt text: The first figure shows that the optimal quantity is increasing concave with quota and decreasing with GHG tax. The second figure shows that the optimal cost is decreasing convex with quota and increasing with GHG tax.

Figure 2 illustrates the effect of the demand variability, as represented by its coefficient of variation σ/μ, on the optimal quantity and cost, for several values of the quota x. For the standard NVP, we remind that the optimal quantity and cost are linear functions of σ (see equations (10) and (11)). Figure 2 shows that this property does not hold for D-NVP, except for $x = 0$ or when x goes to infinity. When $x = 0$, the problem is equivalent to a NVP with underage cost $b = 3$ and overage cost $h + c_d = 6 > b$. It results that the optimal order quantity decreases linearly with σ. When x goes to infinity, the problem is equivalent to a NVP with underage cost $b = 3$ and overage cost $h = 1 < b$. It results that the optimal quantity increases linearly with σ. We also observe that σ has a non-monotonic effect on the optimal quantity, for $x = 25, 50, 100$. When σ is small, the optimal quantity increases with σ to avoid lost sales. When σ is large, the optimal quantity decreases to avoid exceeding the allocated quota. From an application perspective, in a standard NPV, the rule of thumb is "the more variable the demand, the larger the optimal quantity" (when $b > h$). This rule of thumb is no longer valid for D-NVP. This means that managers should be vigilant when working under a cap-and-trade emission regulation since an increased variability in demand can lead to smaller order quantities.

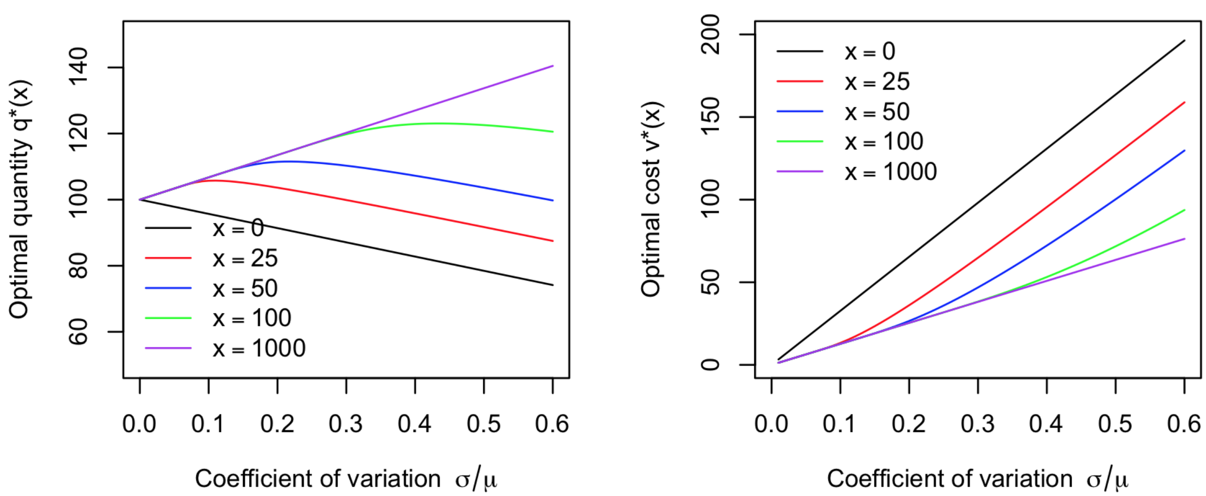

Figure 2: Effect of the demand coefficient of variation on the optimal quantity and cost ($h = 1, b = 3, c_d = 5, r_d = 0$, normally distributed demand with mean $\mu = 100$)

Figure 2 Alt text: The first figure shows that the optimal quantity increases (or decreases) linearly with the coefficient of variation when the quota is large (or small). For intermediate values of the quota, the optimal quantity increases and then decreases with the coefficient of variation. The second figure shows that the optimal cost is increasing convex with the coefficient of variation for all quota values.

In conclusion, we were able to extend the results of the standard NVP with a normal distribution to the D-NVP. We have also shown that the demand variability has a more complex effect in the D-NVP than in the standard NVP.

3.3.3 Discrete demand

Finally we address the case of a discrete demand with probability mass function p, such that $p(s) = 0$ for $s \notin \mathbb{N}$. We have

$$v(x, q) = G(q) + c_d \sum_{s=-\infty}^{q-x-1} (q - x - s)p(s) \tag{12}$$

Let $\Delta v(x, q) = v(x, q + 1) - v(x, q)$. For the standard NVP, we know that $\Delta G(q) = G(q + 1) - G(q) = (h + b)F(q) - b$. It follows that

$$\begin{aligned}
\Delta v(x, q) &= \Delta G(q) + c_d \left\{ \sum_{s=-\infty}^{q-x} (q + 1 - x - s)p(s) - \sum_{s=-\infty}^{q-x-1} (q - x - s)p(s) \right\} \\
&= (h + b)F(q) - b + c_d \sum_{s=-\infty}^{q-x} p(s) \\
&= (h + b)F(q) + c_d F(q - x) - b
\end{aligned}$$

Since $\Delta v(x, q)$ is non-decreasing in q, it is discrete convex in q. Hence the smallest optimal order quantity is

$$q^*(x) = \min \{q \in \mathbb{N} \, : \Delta v(x, q) \geq 0\}$$

and we get the following theorem.

Theorem 4 (Discrete demand) *For the synchronous D-NVP with discrete demand and quota x, the smallest optimal order quantity is*

$$q^*(x) = \min \{q \in \mathbb{N} : \ (h + b)F(q) + c_d F(q - x) \geq b\} . \tag{13}$$

Moreover, we have

$$q^*(x) \leq q^*(x + 1) \leq q^*(x) + 1. \tag{14}$$

Since F is non-decreasing, the optimal quantity can be easily computed, for example by a bisection method in $\mathcal{O}(\log Q)$ with Q an upper-bound on the optimal quantity.

3.4 Simple heuristic policies

Having presented the optimal solution and some of its properties for the synchronous D-NVP, we consider the usefulness of such solutions by comparing them with a few simple and intuitive heuristic policies:

- H_1 : Order $q^*(0)$, the optimal quantity of a standard NVP with underage cost b and overage cost $h + c_d$. This would be optimal if the GHG tax c_d was systematically charged for disposed units, or if there is no quota. This policy is overly conservative and tends to order less than the optimal quantity;

- H_2 : Order $q^*(\infty)$, the optimal quantity of a standard NVP with underage cost b and overage cost h. This would be optimal if the GHG tax was null ($c_d = 0$). The policy is overly risky and tends to order more than the optimal quantity;

- H_3 : Order $q_H(x) = \min\{x + q^*(0), q^*(\infty)\}$. This policy orders the optimal quantity when $x = 0$ or when $x \geq q^*(\infty)$.

We can compare the order quantities of the different policies with Equation (14):

$$q^*(0) \leq q^*(x) \leq q_H(x) \leq q^*(\infty).$$

Figure 3 plots the quantities and costs of these three heuristic policies and the optimal policy. As expected, H_1 performs well for small quotas and poorly for large quotas. It is the opposite for H_2 which performs well for large quotas and poorly for small quotas. Heuristic H_3 performs very well for small or large quotas but is significantly outperformed by the optimal policy for intermediate quotas.

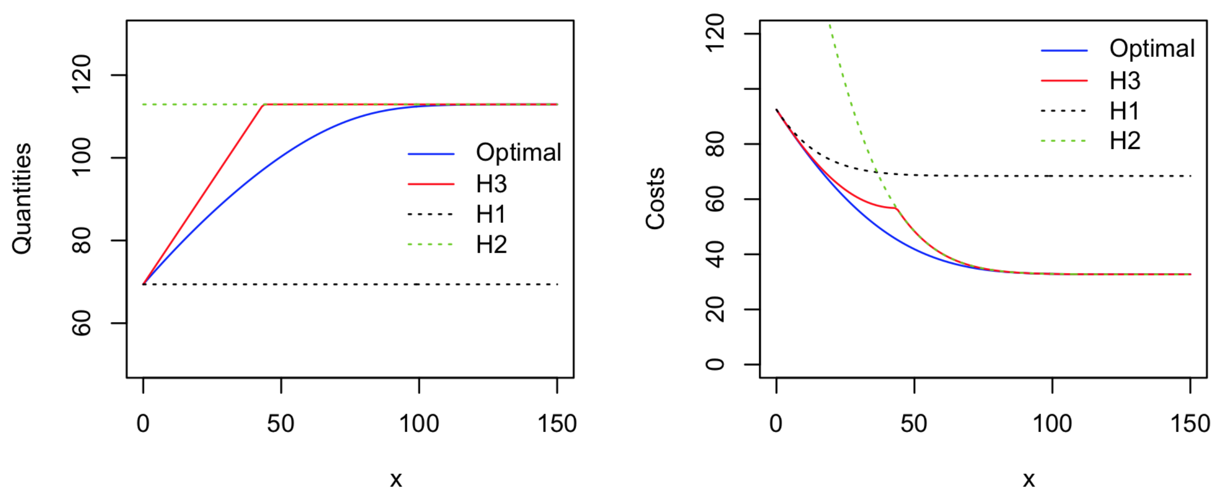

Figure 3: Effect of quota on the optimal quantity and cost ($h = 1, b = 2, c_d = 10, r_d = 0$, normally distributed demand with parameters $\mu = 100, \sigma = 30$)

Figure 3 Alt text: The figure shows that H_1 performs well for small quotas and poorly for large quotas. It is the opposite for H_2 which performs well for large quotas and poorly for small quotas. Heuristic H_3 performs very well for small or large quota but is significantly outperformed by the optimal policy for intermediate quotas.

From a practical point of view, implementing heuristic H_2 is equivalent to consider a situation in which the decision maker would ignore the carbon tax and therefore act as if the GHG quota were unlimited. Therefore, Figure 3 shows that ignoring the carbon tax may lead to large overcost, especially when the GHG quota is small.

4 The asynchronous D-NVP

The synchronous D-NVP presented in the former section is a simplified version of the more ubiquitous case in which the replenishment occurs following the company requirements, while the GHG emmissions are assessed by the regulator, at a different frequency. This general case, which we have called asynchronous D-NVP, is presented and solved in this section.

4.1 Problem description

We consider a D-NVP that is repeated over T time periods. We let D_t be the demand in period $t \in \{1, \cdots, T\}$. We assume that $D_1, \cdots, D_T$ are identically and independently distributed (i.i.d.) with the same distribution as D (introduced in Section 3.1). We also assume that the demand is discrete and bounded by D_{max} in order to be able to compute the optimal policy. The company has a GHG quota x_{max} that is allocated for the whole time horizon and that is available at the begininng of period 1. A GHG tax c_d is charged for each disposed unit exceeding x_{max} at the end of the horizon. If we denote by Q_t the order quantity in period t, the total number of disposed units is $N = \sum_{t=1}^{T}(Q_t - D_t)^+$ and the total GHG tax is $c_d[N - x_{max}]^+$.

Let us denote by X_t the unused quota at the beginning of period t. Variable X_t is random and results from the unused quota, order quantity, and demand realization in the precedent period, with the notable exception of $X_1 = x_{max}$ which is deterministic and represents the cap determined by regulation. We have the following relation between X_{t+1} and X_t :

$$X_{t+1} = [X_t - (Q_t - D_t)^+]^+.$$

It implies that $0 \leq X_t \leq x_{max}$ for $t = 1, \ldots, T$.

A policy π specifies the quantity to order in each period, for each state of the system. At the beginning of period t, the state of the system can be summarized by the unused quota X_t. We denote by $v_t^\pi(x)$ the expected cost under policy π from period t to T if $X_t = x$. Let $v_t^*(x)$ be the optimal expected cost from period t to period T when the unused quota is x at the beginning of period t. We have :

$$v_t^*(x) = \min_\pi v_t^\pi(x).$$

The objective is to determine the policy that minimizes the total expected cost. This problem can be seen as a Markov decision problem with finite state space $\{0, \cdots, x_{max}\}$ and finite action space $\{0, \cdots, D_{max}\}$, since the optimal order quantity does not exceed the maximum demand. By Proposition 4.4.3 in Puterman [1994], there exists a deterministic Markovian policy which is optimal. Hence we can restrict our analysis to deterministic Markovian policies. Therefore the optimal policy can be fully described by $q_t^*(x)$, the optimal order quantity in period t given unused quota x.

For the reader's convenience, we summarize below the additional notations that will be used in the rest of this section.

Parameters

T	Number of periods
D_t	Demand in period t
x_{max}	GHG quota allocated for the whole time horizon
D_{max}	Maximum demand

Decision variables

$q_t^*(x)$	Optimal order quantity in period t when the unused quota is x

Cost functions

$v_t^*(x)$	Optimal expected cost from period t to T if the unused quota is x at the beginning of period t
$v_t(x,q)$	Expected cost from t to T if we order q in period t and follow the optimal policy from $t+1$ to T, with x the remaining quota in t

Note that the deterministic version of this problem is trivial. It is optimal to order exactly the demand and the optimal cost is equal to zero.

4.2 Stochastic dynamic program

In order to compute the optimal policy and cost for the asynchronous D-NVP, we propose in this section a stochastic dynamic program, study its algorithmic complexity and use it to derive some theoretical results.

Let $v_t(x,q)$ be the expected cost from period t to period T when $X_t = x$, we order a quantity q at time t and follow the optimal policy from period $t+1$ to T.

We first set the initial conditions for the stochastic dynamic program. For $x \in [0, x_{max}]$,

$$v_{T+1}^*(x) = 0.$$

Then we have, for $t = 1, \dots, T$, $x = 1, \dots, x_{max}$ and $q = 1, \dots, D_{max}$:

$$v_t(x,q) = v(x,q) + E\left\{v_{t+1}^*([x-(q-D_t)^+]^+)\right\} \quad (15)$$

$$v_t^*(x) = \min_{q\geq 0} \quad v_t(x,q) \quad (16)$$

$$q_t^*(x) = \operatorname*{argmin}_{q\geq 0} \quad v_t(x,q). \quad (17)$$

In Equation (15), $[x-(q-D_t)^+]^+$ represents the unused quota in period $t+1$ with x, D_t, and q respectively the unused quota, the demand, and the order quantity in period t. Remind also that $v(x,q)$, in Equation (15), is the expected cost of the synchronous D-NVP presented in Section 3.1 :

$$v(x,q) = \sum_{s=0}^{D_{max}} p(s)\left\{h(q-s)^+ + b(s-q)^+ + c_d(q-x-s)^+\right\} \quad (18)$$

We can compute the expectation in Equation (15) as follows :

$$E\left\{v^*_{t+1}([x-(q-D)^+]^+)\right\} = \sum_{s=0}^{D_{max}} p(s) v^*_{t+1}([x-(q-s)^+]^+)$$

We now provide some details, in Algorithm 1, for the implementation of the stochastic dynamic program.

Algorithm 1 Stochastic dynamic program

Input: $h, b, c_d, T, D_{max}, q_{max}, x_{max}, v_{max}, p(s)$ for $s = 0, \cdots, D_{max}$
Output: $v^*_t(x)$ and $q^*_t(x)$ for $x = 0, \cdots, x_{max}$ and $t = 1, \cdots, T+1$

$t \leftarrow T+1$
for $x = 0, \cdots, x_{max}$ **do**
  $v^*_t(x) \leftarrow 0$
while $t > 1$ **do**
  $t \leftarrow t+1$
  for $x = 0, \cdots, x_{max}$ **do**
    $v^*_t(x) \leftarrow v_{max}$
    for $q = 0, \cdots, q_{max}$ **do**
      $v \leftarrow 0$
      for $s = 0, \cdots, D_{max}$ **do**
        $v \leftarrow v + p(s)\{h(q-s)^+ + b(s-q)^+ + c_d(q-x-s)^+ + v^*_{t+1}([x-(q-s)^+]^+)\}$
      if $v < v^*_t(x)$ **then**
        $v^*_t(x) \leftarrow v$
        $q^*_t(x) \leftarrow q$

There are four loops in this algorithm and the complexity is in $\mathcal{O}(T \cdot x_{max} \cdot q_{max} \cdot D_{max})$. There is no interest to have an order quantity larger than the demand and we can assume that $q_{max} \leq D_{max}$. It follows that the algorithm complexity is in $\mathcal{O}(T \cdot x_{max} \cdot D^2_{max})$.

Finally, we establish some theoretical results for the optimal cost $v^*_t(x)$.

Theorem 5 *The optimal value function $v^*_t(x)$ is non-increasing in unused quota x and in time t.*

We prove this theorem by induction. Trivially, $v^*_{T+1}(x)$ is non-increasing in x. Assume that $v^*_{t+1}(x)$ is non-increasing in x. From Theorem 1, we know that $v(x,q)$ is non-increasing in x. It follows from Equation (15) that $v_t(x,q)$ is non-increasing in x. So does $v^*_t(x) = \min_{q \geq 0} v_t(x,q)$. On the other hand, we have $[x-(q-D)^+]^+ \leq x$. Hence, by Equation (15), $v_t(x,q) = v(x,q) + E\left\{v^*_{t+1}([x-(q-D_t)^+]^+)\right\} \geq v(x,q) + v^*_{t+1}(x) \geq v^*_{t+1}(x)$ and $v^*_t(x) = \min_{q \geq 0} v_t(x,q) \geq v^*_{t+1}(x)$.

We can also bound the cost of the asynchronous D-NVP as follows. We know that the unused quota X_t at the beginning of period t is between 0 and x_{max} (the GHG quota).

Hence, the cost incurred in period t is between $v^*(x_{max})$ and $v^*(0)$, the optimal costs of the synchronous D-NVP with respectively quotas x_{max} and 0. It follows that

$$T \cdot v^*(x_{max}) \leq v_1^*(x_{max}) \leq T \cdot v^*(0).$$

4.3 Illustration of the optimal policy and cost

In this section, we apply the above stochastic dynamic program to the case of a demand following a Poisson distribution with mean λ, i.e. $P(D = k) = \frac{e^{-\lambda}\lambda^k}{k!}$ for k non-negative integer. Figure 4 illustrates the behavior of the optimal cost and policy as a function of time period t and unused quota x.

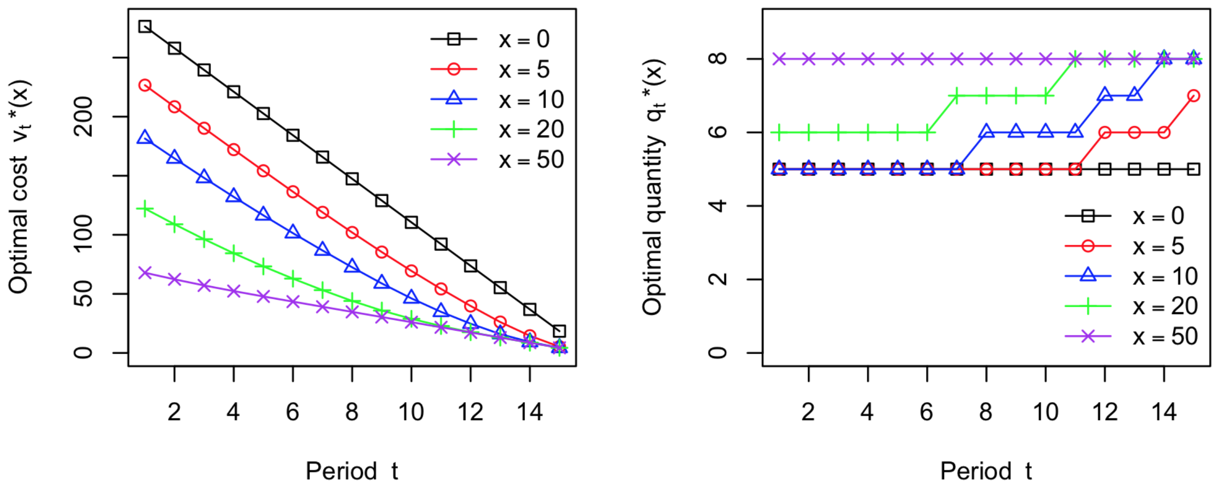

Figure 4: Optimal quantity and cost for an asynchronous D-NVP ($T = 15, h = 1, b = 10, c_d = 10, r_d = 0$, Poisson demand with mean $\lambda = 5$)

Figure 4 Alt text: The first figure shows that the optimal cost is decreasing with time and unused quota. The second figure shows that the optimal quantity is increasing with time and quota.

We observe that the optimal quantity $q_t^*(x)$ is non-decreasing in t. When the number of remaining periods is diminishing, there is less and less reason to keep the unused quota for future periods and it is preferable to increase the order quantity. From a managerial point of view, this means that when approaching the end of the time horizon, the uncertainty reduction allows the decision maker to adopt a riskier policy and order larger quantities (for the same unused quota). We also see that the optimal cost $v_t^*(x)$ is non-increasing and convex in t. Finally, we observe that when the quota is null or very large, the optimal quantity $q_t^*(x)$ is constant and the optimal cost is linear in t. The reason for this is that in both cases, the problem is equivalent to solving T independent NVPs with underage cost b and overage cost h (when x is very large) or overage cost $h + b$ (when $x = 0$).

4.4 Synchronous versus asynchronous D-NVP

In order to assess the benefits of the added flexibility offered by the asynchronous D-NVP over the synchronous case, we compare two types of policies derived from these two problems. In the first policy, a quota x is allocated to each period and can not be used in other periods. In the second policy, a quota Tx is allocated to the whole horizon. On a practical level, a choice between these policies is offered to practitioners in either one of two situations. The first occurs when the emission mechanism between the company and the GHG authority can be negotiated. In the second, when faced to a regulation based on a multi-period horizon, the use of a simple rule of thumb policy is considered as an alternative to an optimal but more complex solution.

More precisely, we consider the two following policies for the problem introduced in Section 4.1 with i.i.d. demands $D_1, \cdots, D_T$.

1. Synchronous Disposal (SD) policy : The problem is divided into T independent periods with a fix quota x for each period. At the end of the period the remaining quota is therefore lost. The optimal cost of a single period is $v^*(x)$, as defined in Section 3. Hence, the cost of SD policy over the T periods is $v^{SD}(x) = Tv^*(x)$. The single-period problem is solved with the algorithm described in Section 3.3.3;

2. Asynchronous Disposal (AD) policy: The quota is allocated for the whole time horizon (T periods), as detailed in Section 4.1. The quota is Tx, i.e. T times the quota for one period under the SD policy. The AD policy is optimal and its expected cost is $v^{AD}(x) = v_1^*(Tx)$. The multi-period problem is solved with the DP algorithm detailed in Section 4.2.

The difference between the costs of the two policys is bounded as follows (see proof in Appendix F).

Theorem 6

$$v^{AD}(x) \leq v^{SD}(x) \leq v^{AD}(x) + Txc_d$$

The quantity Txc_d represents the maximum benefit that can be expected from using the total quota Tx.

In addition, the following theorem establishes that the costs of the two policys are equal when the quota per period x is larger or equal than the optimal quantity of a standard NVP (see proof in Appendix G).

Theorem 7 *If $x \geq q^*(\infty)$, then $v^{AD}(x) = v^{SD}(x) = Tv^*(\infty)$.*

The relative cost increase for using the SD policy instead of the AD policy is given by:

$$\beta = \frac{v^{SD}(x) - v^{AD}(x)}{v^{AD}(x)} = \frac{Tv^*(x) - v_1^*(Tx)}{v_1^*(Tx)}. \tag{19}$$

We conducted a numerical experimentation to compare AD and SD policys over a broad range of scenarios and present here the results. Some of the solution properties presented here are empirical results. Other observations led us to fully demonstrated theoretical results.

First we conduct an exhaustive experimentation over 532 336 instances with the following parameter values:

- Overage cost $h = 1$ (reference cost);
- Underage cost $b \in \{0.1, 0.2, 0.5, 1, 2, 5, 10\}$;
- Carbon tax per unsold item $c_d \in \{0.1, 0.2, 0.5, 1, 2, 5, 10\}$;
- Per-period Poisson demand with mean $\lambda \in \{1, 2, 5, 10, 20, 50, 100\}$;
- Horizon $T \in \{1, \cdots, T_{max}\}$ with $T_{max} = 50$;
- $x \in \{0, \cdots, \lceil \frac{x_{max}}{T} \rceil\}$ with $x_{max} = 350$.

For the 7^3 tuples (b, c_d, λ), we run the DP described in Section 4.2 for an initial total quota of $x_{max} = 350$ and a number of periods $T_{max} = 50$. For each tuple, it provides us the cost of the AD policy for 1 552 sub-problems with $T = 1, \cdots, T_{max}$ and any quota per-period $x \in \{0, \cdots, \lceil \frac{x_{max}}{T} \rceil\}$. We bound x with $\lceil \frac{x_{max}}{T} \rceil$ to avoid any non-integer per-period quota. In the end, we have $7^3 \times 1\,552 = 532\,336$ instances. Here some general observations:

- The relative cost increase β is always non-negative consistently with Theorem 6;
- The relative cost increase β reaches a maximum value of 108% meaning that SD policy sometimes outperforms AD policy by a very large margin;
- As expected, $\beta = 0$ when $T = 1$ or $x = 0$ or $x \geq q^*(\infty)$ (see Theorem 7 for an explanation of the last case);
- The average relative cost increase is 8.83% when we exclude instances with $T = 1$ or $x = 0$ or $x \geq q^*(\infty)$.

To better understand the sensitivity of the solution to the different parameters, a second experimentation phase was performed. To keep the number of experiments reasonable while still reaching meaningful insights, we scrutinized each parameter of interest over a wide range of its values while other parameters are kept within a limited number of values. In the following we shortly discuss the impact of each of the main parameters on the relative cost increase β. Figure 5 illustrates the effect of parameters b, c_d, λ, x and T on the cost of AD and SD policys and relative cost increase β. In each sub-figure, one parameter is varied starting from the following nominal instance : $h = 1, b = 10, c_d = 10, \lambda = 5, x = 2, T = 10$.

Sensitivity to underage cost b

In Figure 5.a, we observe that the absolute costs are increasing in b as expected. Their difference, however, both absolute and, as a result, relative, tends to zero when b tends to zero or to ∞. As a result β is maximal for intermediate values of b.

Proposition 1 *The relative cost increase β goes to 0 either when b goes to 0 or to $+\infty$.*

The proof of this proposition, and those that follow, are provided in Appendix H.

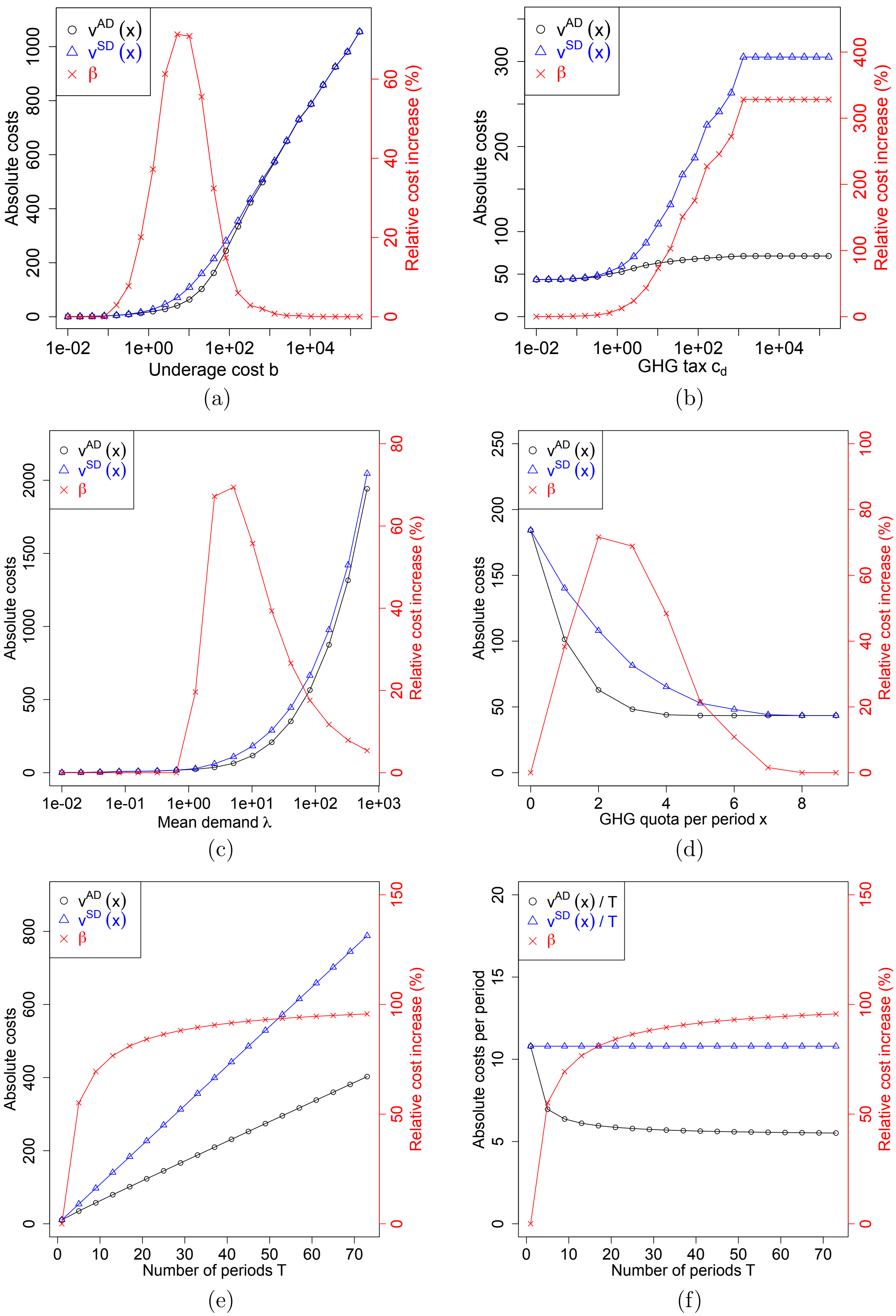

Figure 5: AD policy versus SD policy ($h = 1, b = 10, c_d = 10, \lambda = 5, x = 2, T = 10$)

Figure 5 Alt text: The figure includes 6 graphics, each one representing $v^{AD}(x), v^{SD}(x)$ and β as a function of one of the parameters.

Sensitivity to carbon tax c_d

In Figure 5.b, we observe that the absolute costs are increasing in c_d. The cost of AD policy is increasing much slower than the one of the SD policy. When $c_d = 0$, the two costs are equal to $Tv^*(\infty)$, i.e. the cost of T standard NVP with overage cost h and underage cost b.

With c_d increasing, the optimal quantity $q^*(x)$ under SD policy gradually decreases, until it reaches quota x. When $q^*(x) = x$, there is no incurred disposal cost (see the definition of the disposal cost in Equation (5)). Any further increase of c_d do not induce any additional cost, and therefore the cost $v^{SD}(x)$ remains constant as is shown by the plateau in the right side of the graph.

Sensitivity to mean demand λ

In Figure 5.c, we observe that the effect of λ is very similar to the one of b except that the absolute cost difference does not go to 0 when λ goes to infinity.

Proposition 2 *The relative cost increase β goes to 0 either when λ goes to 0 or to $+\infty$.*

Sensitivity to GHG quota x

Consistently with theorems 1 and 5, Figure 5.d shows that the costs are non-increasing with x. The two costs are equal when $x = 0$ or $x \geq q^*(\infty)$ as shown in Theorem 7. The effect of x on the relative cost increase is summarized in the following proposition.

Proposition 3 *The relative cost increase β goes to 0 either when x goes to 0 or to $+\infty$.*

Sensitivity to horizon length T

When $T = 1$, the two policys are the same and therefore have the same cost. $v^{SD}(x)$ is increasing linearly with T since $v^{SD}(x) = Tv^*(x)$. In Figure 5.e, the cost $v^{AD}(x)$ seems to increase linearly with T but this is, in fact, not the case. To better show this behavior, Figure 5.f compares the costs normalized by period, $v^{SD}(x)/T$ and $v^{AD}(x)/T$. Finally, we observe that β is increasing with T which means that the longer the horizon, the higher the flexibility benefits of the AD policy.

5 Conclusion

In this work, we study a NVP with GHG emissions at the disposal stage, under a cap-and-trade regulation. First, we investigate the synchronous problem, when the GHG quota is allocated to a single period of the NVP. For this problem, we develop a simple algorithm to compute the optimal policy. We show that ignoring the carbon tax may lead to large overcost, especially when the GHG quota is small. We also show that, for a normally distributed demand, in contrast with the classical NVP, the order quantity is non linear (and non monotonic) in the demand standard deviation when the emission quota is limited.

Second, we study an asynchronous version of the problem, when the GHG quota is allocated to several periods of the NVP. The present research lays the foundations of this type of model, which is more in line with the industrial reality where quotas are given per year while replenishment decisions are made at a higher frequency (e.g. daily or weekly). For this problem, we develop an efficient dynamic program to compute the optimal policy. In an extensive numerical study, we observe that the added flexibility offered by an asynchronous disposal policy over a synchronous disposal policy allows for a reduction of costs, specially for long time horizons (the longer the horizon, the higher the flexibility). By dynamically adjusting the level of activity to the remaining GHG emission allowance, companies can reduce their expenses. In parallel, adjusting the order quantity dynamically while taking into account what remains of the emission allowance at the current time point allows to meet more precisely the cap set by the regulator. While this does not necessarily mean a lower level of emissions, the proposed dynamic policy results in a more precise use of the planned quota, and with it a lower variability in emissions. This allows the regulator to plan and deploy more effectively a cap-and-trade emission reduction regulation, and get more predictable results from it.

As an avenue for future research, it would be natural to first address the limitations of the model presented inhere, for example by including GHG emissions on production, sales and returns. In addition, the possibility of trading emission allowances at each period to adjust the current level of emission quota based on the time point and the forecasted demand can be considered. The current model can be further extended by considering the case of multiple product portfolios, as well as the availability of several production and/or disposal technologies, all characterized by their cost and their level of emissions. More generally, considering emissions at the disposal stage in various lot sizing problems other than NVPs is also expected to raise interesting research questions. It would be also of interest to obtain additional results for the asynchronous problem, such as the convexity of the value function.

Acknowledgment

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The authors would like to thank Professor Nabil Absi for his gracious and helpful contribution to the problem statement.

Data availability statement

The data supporting the findings of this study are available within the article.

Appendix

A Equivalence with a problem without GHG reward

From Section 3.1, we have

$$\begin{aligned} v(x,q) &= G(q) + Z(x,q) \\ &= hE(q-D)^+ + bE(D-q)^+ + c_d E\left[(q-D)^+ - x\right]^+ - r_d E\left[x - (q-D)^+\right]^+. \end{aligned} \tag{20}$$

Since $[-y]^+ = y^+ - y$, we can rewrite Equation (4) as

$$Z(x,q) = (c_d - r_d)E[(q-D)^+ - x]^+ + r_d E[(q-D)^+ - x] \tag{21}$$

$$= (c_d - r_d)E[(q-D)^+ - x]^+ + r_d E(q-D)^+ - r_d x \tag{22}$$

Since $[(q-D)^+ - x]^+ = [q-D-x]^+$ when $x \geq 0$, the emission cost can be expressed as

$$Z(x,q) = (c_d - r_d)E(q-x-D)^+ + r_d E(q-D)^+ - r_d x \tag{23}$$

and the total cost as

$$v(x,q) = (h + r_d)E(q-D)^+ + bE(D-q)^+ + (c_d - r_d)E\left[(q-D)^+ - x\right]^+ - r_d x. \tag{24}$$

An instance of the problem is characterized by a vector of parameters $I = (h, b, c_d, r_d, F)$. Remind that we assume that $c_d - r_d \geq 0$. By comparing equations (20) and (24), we conclude that the instance $\tilde{I} = (h + r_d, b, c_d - r_d, 0, F)$ has a total cost $\tilde{v}(x,q) = v(x,q) + r_d x$. Since $r_d x$ is a constant with respect to q, instances I and $\tilde{I}$ have the same optimal quantity and the optimal cost is simply shifted. Hence we can set $r_d = 0$ without loss of generality in the rest of the paper.

B Proof of Lemma 2

It is easy to derive the upper-bound. We have $v^*(x) \leq v^*(0)$ since $v^*(x)$ is non-decreasing in x (see Theorem 1).

It is slightly more involved to derive the lower-bound. Let write $v(x,q)$ as $v(0,q)$ minus some quantity. We have from (1), (3) and (5) that:

$$v(x,q) = E\left[b(D-q)^+ + h(q-D)^+ + c_d(q-D-x)^+\right] \tag{25}$$

$$= E\left[b(D-q)^+ + (h + c_d)(q-D)^+\right] - c_d E\left[(q-D)^+ - (q-D-x)^+\right] \tag{26}$$

$$= v(0,q) - c_d E\left[(q-D)^+ - (q-D-x)^+\right]. \tag{27}$$

As $(q-D)^+ - (q-D-x)^+ \leq x$, it follows that :

$$v(x,q) \geq v(0,q) - c_d x.$$

Then

$$v^*(x) = \min_q v(x,q) \tag{28}$$

$$\geq \min_q v(0,q) - c_d x \tag{29}$$

$$= v^*(0) - c_d x \tag{30}$$

We conclude that $v^*(x) \geq v^*(0) - c_d x$.

C Proof of Lemma 3

Assume in what follows that $x \geq q^*(\infty)$. As $v^*(x)$ is non-increasing in x (see Theorem 1), we have

$$v^*(x) \geq v^*(\infty). \tag{31}$$

From (1), we have

$$v(x, q^*(\infty)) = G(q^*(\infty)) + Z(x, q^*(\infty))$$

On one hand $x \geq q^*(\infty)$ implies that $Z(x, q^*(\infty)) = c_d E\left[q^*(\infty) - D - x\right]^+ = 0$. On the other hand $G(q^*(\infty)) = v^*(\infty)$. Hence we have

$$v(x, q^*(\infty)) = v^*(\infty). \tag{32}$$

(31) and (32) imply that $v(x, q^*(\infty)) \leq v^*(x)$. Hence $q^*(\infty)$ minimizes $v(x, q)$ and we obtain that $q^*(x) = q^*(\infty)$ and $v^*(x) = v^*(\infty)$.

D Bounds on the optimal quantity

Consider a continuous demand. Let show that $q^*(x) \leq q^*(x+1)$. We have

$$\begin{aligned}
\frac{\partial v}{\partial q}(x, q) &= (h+b)F(q) - b + c_d F(q - x) \\
&\geq (h+b)F(q) - b + c_d F(q - (x+1)) = \frac{\partial v}{\partial q}(x, q+1)
\end{aligned}$$

It follows that

$$\begin{aligned}
q^*(x) &= \min\left\{q : \frac{\partial v}{\partial q}(x, q) = 0\right\} \\
&\leq \min\left\{q : \frac{\partial v}{\partial q}(x, q+1) = 0\right\} = q^*(x+1)
\end{aligned}$$

Let us show now that $q^*(x+1) \leq q^*(x) + 1$. We have

$$\begin{aligned}
\frac{\partial v}{\partial q}(x+1, q^*(x)+1) &= (h+b)F(q^*(x)+1) - b + c_d F(q^*(x) - x) \\
&\geq (h+b)F(q^*(x)) - b + c_d F(q^*(x) - x) \\
&= \frac{\partial v}{\partial q}(x, q^*(x)) \geq 0
\end{aligned}$$

which implies that $q^*(x+1) \leq q^*(x) + 1$.

E Specific distributions

E.1 Exponential distribution

Assume that the demand follows an exponential distribution with rate λ, that is $F(s) = 1 - \exp(-\lambda s)$ and $f(s) = \lambda \exp(-\lambda s)$ for $s \geq 0$.

Theorem 8 (Exponential distribution) *The expected costs are given by :*

$$G(q) = h\left(q - \frac{1}{\lambda}\right) + (h+b)\frac{e^{-\lambda q}}{\lambda},$$
$$Z(x,q) = c_d\left\{(q-x)^+ - \frac{1-e^{-\lambda(q-x)^+}}{\lambda}\right\}.$$

Moreover, the optimal quantity is given by

$$q^*(x) = \begin{cases} \frac{1}{\lambda}\ln\left(\frac{(h+b)+c_d e^{\lambda x}}{h+c_d}\right) & \text{if } 0 \le x < q^*(\infty) \\ q^*(\infty) & \text{if } x \ge q^*(\infty) \end{cases}$$

where

$$q^*(\infty) = \frac{1}{\lambda}\ln\left(\frac{h+b}{h}\right).$$

We now prove this theorem. We first compute

$$E(D-q)^+ = \int_q^\infty (t-q)\lambda e^{-\lambda t}dt = \frac{e^{-\lambda q}}{\lambda}$$

and

$$E[(q-x)^+ - D]^+ = \int_0^{(q-x)^+} (q-t)\lambda e^{-\lambda t}dt = (q-x)^+ - \frac{1-e^{-\lambda(q-x)^+}}{\lambda}.$$

Then, with equations (3) and (23), we have

$$G(q) = h(q-\mu) + (h+b)pE(D-q)^+ = h\left(q-\frac{1}{\lambda}\right) + (h+b)\frac{e^{-\lambda q}}{\lambda}$$

and

$$\begin{aligned} Z(x,q) &= c_d E[(q-x)^+ - D]^+ \\ &= c_d\left\{(q-x)^+ - \frac{1-e^{-\lambda(q-x)^+}}{\lambda}\right\}. \end{aligned}$$

We have

$$\begin{aligned} \frac{\partial v}{\partial q}(x,q) &= pF(q) + c_d F(q-x) - b \\ &= \begin{cases} (h+b)F(q) - b & \text{if } 0 \le q \le x \\ (h+b)F(q) + c_d F(q-x) - b & \text{if } q \ge x \end{cases} \\ &= \begin{cases} (h+b)\left(1-e^{-\lambda q}\right) - b & \text{if } 0 \le q \le x \\ (h+b)\left(1-e^{-\lambda q}\right) + c_d\left(1-e^{-\lambda(q-x)}\right) - b & \text{if } q \ge x \end{cases} \\ &= \begin{cases} -(h+b)e^{-\lambda q} + h & \text{if } 0 \le q \le x \\ -e^{-\lambda q}\left(h+b+c_d e^{\lambda x}\right) + (h+c_d) & \text{if } q \ge x \end{cases} \end{aligned}$$

Moreover

$$\begin{aligned}
& \frac{\partial v}{\partial q}(x,x) < 0 \\
\Leftrightarrow \quad & -(h+b)e^{-\lambda x} + h < 0 \\
\Leftrightarrow \quad & x < q^*(\infty) := \frac{1}{\lambda}\ln\left(\frac{h+b}{b}\right)
\end{aligned}$$

We can then distinguish two cases.

Case 1 : $x < q^*(\infty)$ The partial derivative is continuous and strictly increasing in q. Hence, the optimal quantity is the unique solution of

$$\begin{aligned}
& \frac{\partial v}{\partial q}(x,q) = 0 \\
\Leftrightarrow \quad & -e^{-\lambda q}\left(h+b+c_d e^{\lambda x}\right) + (h+c_d) = 0 \\
\Leftrightarrow \quad & q = \frac{1}{\lambda}\ln\left(\frac{h+b+c_d e^{\lambda x}}{h+c_d}\right)
\end{aligned}$$

Case 2 : $x \geq q^*(\infty)$ The optimal quantity is the unique solution of

$$\begin{aligned}
& \frac{\partial v}{\partial q}(x,q) = 0 \\
\Leftrightarrow \quad & -(h+b)e^{-\lambda q} + h = 0 \\
\Leftrightarrow \quad & q = \frac{1}{\lambda}\ln\left(\frac{h+b}{h}\right) = q^*(\infty)
\end{aligned}$$

Finally we have

$$q^*(x) = \begin{cases} \frac{1}{\lambda}\ln\left(\frac{h+b+c_d e^{\lambda x}}{h+c_d}\right) & \text{if } 0 \leq x < q^*(\infty) \\ q^*(\infty) & \text{if } x \geq q^*(\infty) \end{cases}$$

where $q^*(\infty) = \frac{1}{\lambda}\ln\left(\frac{h+b}{h}\right)$. □

E.2 Uniform distribution

Assume now that the demand follows a continous uniform distribution $\mathcal{U}(a_1,a_2)$. For $s \in [a_1,a_2]$, we have $F(s) = (s-a_1)/(a_2-a_1)$.

Theorem 9 (Uniform distribution) *For $q \in [a_1,a_2]$, the expected costs are given by*

$$G(q) = \frac{h(q-a_1)^2 + b(a_2-q)^2}{2(a_2-a_1)},$$

$$Z(x,q) = c_d\frac{\left([q-x-a_1]^+\right)^2}{2(a_2-a_1)}.$$

Moreover, the optimal quantity is given by

$$q^*(x) = \begin{cases} a_1 + \frac{b(a_2-a_1)+c_d x}{h+b+c_d} & \text{if } 0 \leq x \leq q^*(\infty) - a_1 \\ q^*(\infty) & \text{if } x \geq q^*(\infty) - a_1 \end{cases}$$

where

$$q^*(\infty) = a_1 + \frac{b(a_2 - a_1)}{h + b}.$$

We now prove this theorem.

$$F(s) = \begin{cases} 0 & \text{if } s < a_1 \\ \frac{s-a_1}{a_2-a_1} & \text{if } a_1 \leq s \leq a_2 \\ 1 & \text{if } s \geq a_1 \end{cases}$$

$$f(s) = \begin{cases} \frac{1}{a_2-a_1} & \text{if } a_1 \leq s \leq a_2 \\ 0 & \text{else} \end{cases}$$

We first compute

$$E(D-q)^+ = \int_q^{a_2} \frac{t-q}{a_2-a_1} dt = \frac{(a_2-q)^2}{2(a_2-a_1)}$$

and

$$\begin{aligned} E[(q-x)^+ - D]^+ &= \int_0^{(q-x)^+} f(t)dt \\ &= \begin{cases} 0 & \text{if } q-x < a_1 \\ \int_{a_1}^{q-x} \frac{q-t}{a_2-a_1} dt & \text{if } a_1 \leq q-x \leq a_2 \end{cases} \\ &= \begin{cases} 0 & \text{if } q-x < a \\ \frac{(q-x-a_1)^2}{2(a_2-a_1)} & \text{if } a_1 \leq q-x \leq a_2 \end{cases} \\ &= \frac{\left([q-x-a_1]^+\right)^2}{2(a_2-a_1)} \end{aligned}$$

Then, with equations (3) and (23), we have

$$\begin{aligned} G(q) = h(q-\mu) + (h+b)E(D-q)^+ &= h\left(q - \frac{a_1+a_2}{2}\right) + h + b\frac{(a_2-q)^2}{2(a_2-a_1)} \\ = \frac{h(q-a_1)^2 + b(a_2-q)^2}{2(a_2-a_1)} \end{aligned}$$

and

$$\begin{aligned} Z(x,q) &= c_d E[(q-x)^+ - D]^+ \\ &= c_d \frac{\left([q-x-a_1]^+\right)^2}{2(a_2-a_1)}. \end{aligned}$$

□

E.3 Normal distribution

We have

$$\begin{aligned} E(Z-z)^+ &= \int_z^{\infty} (t-z)\phi(x)dt \\ &= \int_z^{\infty} \frac{t}{\sqrt{2\pi}} \exp(-\frac{t^2}{2})dt - z\int_z^{\infty} \phi(t)dt \\ &= \left[-\frac{1}{\sqrt{2\pi}} \exp(-\frac{t^2}{2}) \right]_z^{+\infty} - z(1-\Phi(z)) \\ &= \phi(z) + z\Phi(z) - z \end{aligned}$$

As $(Z-z) = (Z-z)^+ - (z-Z)^+$, we have

$$\begin{aligned} E(Z-z)^+ &= E(z-Z)^+ - E(Z-z) \\ &= \phi(z) + z\Phi(z) - z - \mu + z \\ &= \phi(z) + z\Phi(z) \end{aligned}$$

Let $z = \frac{q-\mu}{\sigma}$ and $w = \frac{q-x-\mu}{\sigma} = z - \frac{x}{\sigma}$. It follows that

$$\begin{aligned} G(q) &= h(q-\mu) + (h+b)E(D-q)^+ \\ &= h\sigma z + (h+b)\sigma(\phi(z) + z\Phi(z) - z) \\ &= \sigma \left\{ (h+b)[\phi(z) + z\Phi(z)] - bz \right\} \end{aligned}$$

and

$$\begin{aligned} Z(x,q) &= c_d E(q-x-D)^+ \\ &= c_d \sigma E(w-Z) \\ &= c_d \sigma \left[\phi(w) + w\Phi(w) \right] \end{aligned}$$

Finally

$$\begin{aligned} v(x,q) &= G(q) + Z(x,q) \\ &= \sigma \left\{ (h+b)[\phi(z) + z\Phi(z)] - bz + c_d \left[\phi(w) + w\Phi(w) \right] \right\} \end{aligned}$$

We can also write that

$$\frac{v(x,q)}{\sigma} = (h+b)\phi(z) + c_d\phi(w) + z[(h+b)\Phi(z) + c_d\Phi(w) - b] - c_d\frac{x}{\sigma}\Phi(w).$$

Consider the optimal quantity $q^*(x)$, the normalized optimal quantity $z^*(q) = (q^*(x) - \mu)/\sigma$ and $w^*(x) = z^*(x) - \frac{x}{\sigma}$. We have

$$\begin{aligned} &(h+b)F(q^*(x)) + c_d F(q^*(x) - x) = b \\ \Leftrightarrow &(h+b)\Phi(z^*(x)) + c_d\Phi(w^*(x)) = b. \end{aligned} \tag{33}$$

It follows that

$$
\begin{aligned}
v^*(x) &= v(x, q^*(x)) \\
&= \sigma[(h+b)\phi(z^*(x)) + c_d\phi(w^*(x))] - c_d x\Phi(w^*(x)) \\
&= \sigma(h+b)\phi(z^*(x)) + c_d[\sigma\phi(w^*(x)) - x\Phi(w^*(x))]. \qquad (34)
\end{aligned}
$$

When x goes to infinity, $w^*(x)$ goes to $-\infty$, $\phi(w^*(x))$ to 0 and $\Phi(w^*(x))$ to 0. It follows from equations (33) and (34) that

$$
\begin{aligned}
z^*(x) &\underset{x\to+\infty}{\longrightarrow} z^*(\infty) = \Phi^{-1}\left(\frac{b}{b+h}\right), \\
v^*(x) &\underset{x\to+\infty}{\longrightarrow} \sigma(h+b)\phi(z^*(\infty)).
\end{aligned}
$$

F Proof of Theorem 6

The lower bound is simple to establish. The AD contract corresponds to the optimal policy for the asynchronous D-NVP defined in Section 4.1 while the SD contract corresponds to a heuristic policy for the same problem. It follows that $v^{AD}(x) \leq v^{SD}(x)$.

To establish the upper bound, we need the following lemma that generalizes Lemma 2.

Lemma 4

$$v^{AD}(x) \geq v^{AD}(0) - Txc_d$$

Proof: Under AD contract and some policy π, denote by Q_t^π the order quantity in period t and by Y_t^π the quota that will be consumed in period t.

Let write $v^{AD}(x)$ as $v^{AD}(0)$ minus some quantity:

$$
\begin{aligned}
v_\pi^{AD}(x) &= E\left[\sum_{t=1}^{T} b(D_t - Q_t^\pi)^+ + h(Q_t^\pi - D_t)^+ + c_d(Q_t^\pi - D_t - Y_t^\pi)^+\right] \\
&= E\left[\sum_{t=1}^{T} b(D_t - Q_t^\pi)^+ + (h+b)(Q_t^\pi - D_t)^+\right] \\
&\quad - c_d E\left[\sum_{t=1}^{T}((Q_t^\pi - D_t)^+ - (Q_t^\pi - D_t - Y_t^\pi)^+)\right] \\
&= v_\pi^{AD}(0) - c_d E\left[\sum_{t=1}^{T}((Q_t^\pi - D_t)^+ - (Q_t^\pi - D_t - Y_t^\pi)^+)\right] \\
&\geq v_\pi^{AD}(0) - c_d E\left[\sum_{t=1}^{T} Y_t^\pi\right] \qquad (35) \\
&\geq v_\pi^{AD}(0) - Txc_d. \qquad (36)
\end{aligned}
$$

Inequality (35) follows from $((Q_t^\pi - D_t)^+ - (Q_t^\pi - D_t - Y_t^\pi)^+) \leq Y_t^\pi$. Inequality (36) follows from $\sum_{t=1}^{T} Y_t^\pi \leq Tx$ (the total consumed quota is smaller than the total available quota Tx).

Hence

$$\begin{aligned}v^{AD}(x) &= \min_\pi v_\pi^{AD}(x)\\ &\geq \min_\pi v_\pi^{AD}(0) - Txc_d\\ &= v^{AD}(0) - Txc_d.\end{aligned}$$

We conclude that $v^{AD}(x) \geq v^{AD}(0) - Txc_d$. □

Using Lemma 4 and the fact that $v^{SD}(x) \leq v^{SD}(0)$, we obtain

$$v^{SD}(x) - v^{AD}(x) \leq v^{SD}(0) - v^{AD}(0) + Txc_d.$$

As $v^{SD}(0) = v^{AD}(0)$, we conclude that $v^{SD}(x) - v^{AD}(x) \leq Txc_d$.

G Proof of Theorem 7

Assume in all this proof that $x \geq q^*(\infty)$. For the SD contract, Lemma 3 implies that

$$v^{SD}(x) = Tv^*(x) = Tv^*(\infty). \tag{37}$$

For the AD contract, we have $v^{AD}(x)$ that is non-increasing in x (see Theorem 5). Hence :

$$v^{AD}(x) \geq v^{AD}(\infty) = Tv^*(\infty) \tag{38}$$

We have also from Theorem F that :

$$v^{SD}(x) \geq v^{AD}(x) \tag{39}$$

The three equations (37), (38) and (39) imply that $v^{SD}(x) \geq v^{AD}(x) \geq v^{SD}(x)$ which implies that $v^{AD}(x) = v^{SD}(x)$.

H Proof of sensitivity results

Effect of b

When b is sufficiently small, the optimal quantity is zero for the two contracts, which implies that the cost of both contracts is $b \cdot T \cdot E(X) = b \cdot T \cdot \lambda$ and therefore β is equal to zero. Hence, when b is sufficiently small, we have $\beta = 0$.

From the definition of β given in (19) and Theorem 6, we have

$$0 \leq \beta \leq \frac{Txc_d}{v^{AD}(x)}. \tag{40}$$

When b goes to infinity, $v^{AD}(x)$ goes to infinity while Txc_d remains constant. It follows that β goes to 0 when b goes to infinity.

Effect of λ

The proof is the same as for the effect of b. It suffices to replace b by λ in the above proof.

Effect of x

When $x = 0$, the two contracts have no quota and $v^{AD}(0) = v^{SD}(0) = Tv^*(0)$ which implies that $\beta = 0$. When $x \geq q^*(\infty)$, we have from Theorem 7 that $v^{AD}(x) = v^{SD}(x)$ which implies that $\beta = 0$.

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